SIGNATURE EFFECTS IN TWO QUASI-PARTICLE ROTATIONAL BANDS OF ODD-ODD DEFORMED NUCLEI

Ph.D Proposal

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I. INTRODUCTION

The possibility of the nonspherical nuclear field was first proposed by Rainwater [1] which gives rise to the concept of deformed nuclei. This idea of deformation led us towards the concept of nuclear angular momentum which could be generated by pure single particle motion plus pure collective motion or the combination of both. The competition between single particle motion and collective motion led us to the observation of very complex spectrum-single particle states plus rotational states. The progress in the spectroscopic techniques and the development of large arrays of γ-detectors such as Gammasphere, Euroball and INGA etc. help us to observe various phenomena such as back-bending, band-crossing, band termination, signature splitting, and signature inversion etc. in rotating nuclei. The origin of these phenomena can be located if one have sufficient amount of reliable data. In the past decade so much trustworthy data has been accumulated which provide a good platform for testing nuclear models and theories.

Due to the advent of large gamma detector arrays and sophisticated techniques for data analysis, now we have large amount of data for two-quasiparticle (2qp) states in deformed nuclei. In deformed axially symmetric odd-odd nuclei, the projections of intrinsic angular momenta of two valence particles on the symmetric axis can couple either in parallel or antiparallel fashion, which leads to the two values of band-head spin as $K_+ = \Omega_p + \Omega_n$ and $K_- = \left|\Omega_p - \Omega_n\right|$, where Ω_p and Ω_n are the projections of the angular momenta of a proton and a neutron respectively on the nuclear symmetry axis. The residual *n-p* interaction thought to play a crucial role in the ordering of these singlet (K_-) and triplet energy states (K_+) [2-4]. In order to fix the energy ordering of singlet and triplet states for odd –odd and even-even nuclei, empirical rules were devised by Gallagher & Moszkowski-known as Gallagher-Moszkowski (GM) rules [5,6]. According to these rules, the triplet state (a state having parallel coupling of spins) always lie lower in energy than the singlet state (a state having antiparallel coupling of spins) in odd-odd nuclei but reverse is the situation in case of even–even nuclei i.e. singlet state lie lower in energy than the triplet state. The difference between the energy of singlet and triplet states is known as GM splitting. An additional shift of odd spin members with respect to even spin members has also been observed for $K = 0$ bands $\left(\Omega_p = \Omega_n\right)$ [7]. This shift observed in $K = 0$ bands is known as Newby shift and play a vital role in the signature effects.

We know that the signature quantum number is related to the invariance of the wave function under rotation by an angle π about an axis perpendicular to symmetry axis. This invariance introduces a phase factor $(-1)^{I+K}$ in the wave function and hence in the energy expression. This phase factor alternates the sign for successive values of I, as a result of which one ΔI=1 band get splitted into two ΔI=2 bands connected by E2 transitions. This splitting of one ΔI=1 band into two ΔI=2 sequences is known as signature splitting and these two signature branches are distinguished from each other by a signature quantum number α. In ΔI=2 rotational

bands, the two signature branches are usually not equivalent energetically. Due to the Coriolis force acting on the valence particles, one of them called favored, lies lower in energy than the other branch, called unfavoured. Whenever an expected favored signature branch becomes unfavoured at higher spins i.e. a signature branch, which is expected to be lower in energy, becomes higher in energy, then it is called signature inversion [8-10].

In the present proposal, we focus our study on the theoretical explanation of signature effects (signature splitting & signature inversion) observed in 2qp rotational bands of odd-odd nuclei in A= 160 mass region using Particle plus Rotor Model (TQPRM).

II. LITERATURE REVIEWS

Since last two decades, signature effects observed in 2qp and multi-quasiparticle states has attracted a lot of attention of theoretician as well as of experimentalists. Although different explanations for these signature effects observed in odd-odd nuclei are available in literature, but the universal cause for these phenomena is still an unresolved problem. The present proposal will be one of the important steps in fixing the universal cause of signature effects observed in the 2qp bands of odd-odd nuclei. In Table 1, we present various approaches existing in literature for the explanation of signature effects in odd- odd nuclei in the A=160 mass region. From Table 1, it is clear that most of the work on signature effects using Particle Rotor Model approach has been done only over the period of one decade. **But no quantitative calculations, which incorporates the Coriolis mixing and theoretical residual interactions collectively, for the explanation of signature effects has been done so far. This is the principal motivation behind the present proposal.**

Table 1

References	Theoretical Approach	Nuclides Studied	Conclusions
Jain *et.al.* [11] Phys. Rev.C40, 432-444(1989).	Two particle plus rotor model (axially symmetric) + Empirical residual *n-p* interaction	^{160}Ho	Coriolis band mixing play major role in **signature inversion** in A=160 mass region
Jain *et.al.* [12] Phys. Lett. B277, 233-37(1992).		^{168}Ho,^{176}Lu ^{182}Ta,^{182}Re	
Semmes *et.al.* [13] AIP proceeding,(1991)	Two Particle plus rotor model (axial symmetric) + Theoretical residual *n-p* interaction	^{152}Eu	Residual n-p interaction play a crucial role in **signature inversion** in (without Coriolis band mixing)
Riedinger *et.al.* [10] Nucl. Phys. 38,251-261(1997)		^{164}Tm	Appropriate n-p interaction is very effective to explain **signature inversion (without Coriolis band mixing)**
Semmes *et.al.* [15] AIP proceeding,(1990)	Two particle plus triaxial rotor model	^{152}Eu	γ-deformation play a major role in explaining **signature inversion**
Bengtsson *et.al.* [8] Nucl. Phys. A415, 189-214(1984).		^{154}Tb,^{156}Eu	

III. DESCRIPTION OF PROPOSED APPROACH TO STUDY SIGNATURE EFFECTS

The Particle Plus Rotor Model (PPRM) approach provides an alternative tool for the calculation of nuclear properties at high angular momentum. Since PPRM formulation is in terms of angular momentum, a physical observable in the experiments, so direct comparison with the experimental data can be made. In the present proposal, we focus our study on the theoretical explanation of signature effects observed in 2qp bands of odd-odd nuclei in A= 160 mass region using TQPRM approach.

We will include theoretical residual interaction (V_{np}) in the earlier version of TQPRM [11] which will give us a complete theoretical TQPRM model for the explanation of signature effects. The theoretical estimates of residual interaction will also explore some important information about the *n-p* interaction.

Objectives

- Formulation of theoretical model for the calculation of the band-head energies. This model will incorporate various corrections such as Coriolis and two state mixing on the band-head energies and hence will give better estimates of band-head energies as compared to the results available in literature [16, 17]

- Development of theoretical Two Quasi-particle Plus Rotor Model (TQPRM) for the explanation of signature effects in odd-odd nuclei in the A=160 mass region. We will include theoretical residual interactions in the earlier model by Jain *et al.*[11].

Hypothesis to be tested

Particle rotor calculations with appropriate residual interaction are assumed to be major cause for observed signature effects in odd-odd nuclei. Thus present proposal will confirm the role of residual interactions in the theoretical explanation of signature effects.

VII METHODOLOGY

In the present proposal, we will use TQPRM for the explanation of signature effects in 2qp bands observed in A=160 mass region. The total Hamiltonian of the system in the frame work of TQPRM under adiabatic conditions is divided into two parts; the intrinsic and the rotational parts, and may be written as

$$H = H_{\text{int}} + H_{rot} \tag{1}$$

The intrinsic part of Hamiltonian (H_{int}) further consist of a deformed axially symmetric average field calculated using Nilsson model (H_{av}), a short range pairing interaction (H_{pair}), a vibrational contribution arising from photon-phonon admixer (H_{Vib}), a short range neutron proton residual interaction (V_{np}).This model is formulated by assuming that nucleus remain in its vibrational ground state so that the contributions due to H_{Vib} is very small [11,15].

$$H_{\text{int}} = H_{av} + H_{pair} + H_{vib} + V_{np} \tag{2}$$

For an axially symmetric rotor, the second part of total Hamiltonian (given by equation 1) may be written as

$$H_{rot} = H_{rot}^{0} + H_{cor} + H_{ppc} + H_{irrot} \tag{3}$$

where

$$H_{rot}^{0} = \frac{\hbar^2}{2\Im}\left(I^2 - I_z^2\right) \tag{4}$$

$$H_{cor} = -\frac{\hbar^2}{2\Im}\left(I_+ j_- - I_- j_+\right) \tag{5}$$

$$H_{ppc} = \frac{\hbar^2}{2\Im}\left(j_p^+ j_n^- - j_p^- j_n^+\right) \tag{6}$$

$$H_{irrot} = \frac{\hbar^2}{2\Im}\left[\left(j_p^2 - j_{pz}^2\right) + \left(j_n^2 - j_{nz}^2\right)\right] \tag{7}$$

where $\Im$ denote the moment of inertia with respect to the rotation axis. H_{rot}^{0} represent purely rotational part of axially symmetric even-even core, H_{cor} is the contribution from coupling of particles with core (Coriolis coupling) and known as the Rotor-Particle Coupling (RPC) term, which can couple two states satisfying the condition $\Delta K = 1$, H_{ppc} is contribution corresponding to particle-particle coupling which will contribute for $\Delta K = 0$, and H_{irrot} is the intrinsic contributions, known as irrotational part. The total angular momentum $I = R + j$, where R, j are

angular momentum associated with rotation of core and particles ($j = j_p + j_n$) respectively. The $I_\pm = I_x \pm iI_y$, $j_\pm = j_x \pm ij_y$ are the ladder operators.

The wave function corresponding to Hamiltonian (given by eq.1) when ($K \neq 0$) can be written as the product of intrinsic wave function ($\left|K,\alpha\right\rangle = \left|\rho_p\Omega_p\right\rangle\left|\rho_n\Omega_n\right\rangle$) and the Wigner function ($D^I_{MK}\left(\theta,\phi,\varphi\right)$) which is basis state of a rotating system.

$$\left|IMK,\alpha=\rho_n\rho_p\right\rangle = \sqrt{\frac{2I+1}{16\pi^2\left(1+\delta_{k0}\right)}}\left\{D^I_{MK}\left|K,\alpha\right\rangle + \left(-1\right)^{I+K} D^I_{M-K}R_i\left|K,\alpha\right\rangle\right\} \tag{8}$$

where α characterize the configuration of unpaired particle ($\alpha = \rho_n\rho_p$) such that

$$H_{av}\left|K,\alpha\right\rangle = \left(\varepsilon_n + \varepsilon_p\right)\left|K,\alpha\right\rangle \tag{9}$$

$\left(\varepsilon_n + \varepsilon_p\right)$ are the quasiparticle energies corresponding to neutron and proton and will be calculated using average deformed field [18,19]. K is the projection of intrinsic angular momentum to symmetric axis. The rotation operator ($R_i = e^{-i\pi J_y}$) representing the rotation of nucleus by π about an axis perpendicular to the symmetry axis. As the consequence of symmetry of average field (axial symmetry) projection of particles angular momentum can couple either parallel ($K_+ = \left|\Omega_p + \Omega_n\right| = \left|k_p + k_n\right|$) or anti-parallel ($K_- = \left|\Omega_p - \Omega_n\right| = \left|k_p - k_n\right|$) fashion. These two band heads (K_+, K_-) are degenerate, but the presence of residual *n-p* interaction (V_{np}) introduce an energy splitting between these two band-heads.

The total wave function corresponding to Hamiltonian (given by eq.1) for $K = 0$ rotational bands can be written as the product of rotational and intrinsic wave function.

$$\left|IMK=0,\alpha=\rho_n\rho_p\right\rangle = \sqrt{\frac{2I+1}{32\pi^2}}\left\{D^I_{M0}\left(\begin{array}{l}\frac{1}{\sqrt{2}}\left(\left|\rho_p k\right\rangle\left|\rho_n -k\right\rangle - \left|\rho_p -k\right\rangle\left|\rho_n k\right\rangle\right) + \\ \left(-1\right)^I R_i \frac{1}{\sqrt{2}}\left(\left|\rho_p k\right\rangle\left|\rho_n -k\right\rangle - \left|\rho_p -k\right\rangle\left|\rho_n k\right\rangle\right)\end{array}\right)\right\} \tag{10}$$

The Eigen value of rotation operator (R_i) corresponding to single particle wave function ($\left|K=0,\alpha\right\rangle$) is given by $J_\alpha = \pm 1$ such that the modified wave function corresponding to total Hamiltonian for ($\left|K=0,\alpha\right\rangle$ can be written as

$$\left|IMK=0,\alpha=\rho_n\rho_p\right\rangle=\sqrt{\frac{2I+1}{32\pi^2}}\left\{D^I_{M0}\left[1+(-1)^I J_\alpha\right]\frac{1}{\sqrt{2}}\left(\left|\rho_p k\right\rangle\left|\rho_n -k\right\rangle-\left|\rho_p -k\right\rangle\left|\rho_n k\right\rangle\right)\right\} \quad (11)$$

The value of total wave function is always non zero ($\left|IMK=0,\alpha=\rho_n\rho_p\right\rangle\neq 0$) and by using possible value of $J_\alpha=\pm1$ permit us to write two different wave function corresponding to even nuclear spin ($I=0,2,4..$) (eq. 12 given below) and other corresponding to odd spin ($I=1,3,4..$) (eq. 13 given below).

$$\left|IMK=0,\alpha=\rho_n\rho_p,J_\alpha=+1\right\rangle=\sqrt{\frac{2I+1}{16\pi^2}}\left\{D^I_{M0}\left(\frac{1}{\sqrt{2}}\left(\left|\rho_p k\right\rangle\left|\rho_n -k\right\rangle-\left|\rho_p -k\right\rangle\left|\rho_n k\right\rangle\right)\right)\right\} \quad (12)$$

$$\left|IMK=0,\alpha=\rho_n\rho_p,J_\alpha=-1\right\rangle=\sqrt{\frac{2I+1}{16\pi^2}}\left\{D^I_{M0}\left(\frac{1}{\sqrt{2}}\left(\left|\rho_p k\right\rangle\left|\rho_n -k\right\rangle+\left|\rho_p -k\right\rangle\left|\rho_n k\right\rangle\right)\right)\right\} \quad (13)$$

Thus $K=0$ rotational band can be broken up into two branches one corresponding to $J_\alpha=+1$ and another having $J_\alpha=-1$.

Calculation of Matrix Elements

The matrix elements corresponding to various terms given in equations (4) to (7) in basis function ($\left|IMK\alpha\right\rangle$) are given below:

Matrix elements of Rotational term (eq.4)

$$\left\langle IMK\alpha\left|H^0_{rot}\right|IMK'\alpha'\right\rangle=\left\langle IMK\alpha\left|\frac{\hbar^2}{2\Im}\left(I^2-I_z^2\right)\right|IMK'\alpha'\right\rangle=\delta_{KK'}\delta_{\alpha\alpha'}\frac{\hbar^2}{2\Im}\left[I\left(I+1\right)-K^2\right] \quad (14)$$

Matrix elements of Coriolis Coupling (RPC) term (eq.5)

$$\begin{aligned}\left\langle IMK\alpha\left|H_{cor}\right|IMK'\alpha'\right\rangle&=\left\langle IMK\alpha\left|-\frac{\hbar^2}{2\Im}\left(I_+ j_- - I_- j_+\right)\right|IMK'\alpha'\right\rangle\\ &=-\frac{\hbar^2}{2\Im}\begin{bmatrix}\delta_{KK'-1}\sqrt{\left(I+K'\right)\left(I-K'+1\right)}\left\langle K'-1\alpha\left|j_-\right|K'\alpha'\right\rangle+\\ \delta_{KK'+1}\sqrt{\left(I-K'\right)\left(I+K'+1\right)}\left\langle K'+1\alpha\left|j_+\right|K'\alpha'\right\rangle+\\ \delta_{K=1,K'=0}\frac{1}{2\sqrt{2}}\left[\left(1+(-1)^I J_{\alpha'}\right)^2\right]\sqrt{I\left(I+1\right)}\left\langle K\alpha\left|j_+\right|K'=0\alpha' J_{\alpha'}\right\rangle+\\ \delta_{K=0,K'=1}\frac{1}{2\sqrt{2}}\left[\left(1+(-1)^I J_\alpha\right)^2\right]\sqrt{I\left(I+1\right)}\left\langle K=0\alpha\left|j_+\right|K'\alpha'\right\rangle\end{bmatrix}\end{aligned} \quad (15)$$

The matrix element corresponding to $K=0\,\&\,K'=0$ will vanishes, because of the terms $\delta_{0,-1}\,\&\,\delta_{0,1}$ appearing in $\left\langle D^I_{M0}\left|I_-\right|D^I_{M0}\right\rangle\,\&\,\left\langle D^I_{M0}\left|I_+\right|D^I_{M0}\right\rangle$ respectively.

Matrix elements of Particle-Particle Coupling (PPC) term (eq.6)

$$\langle IMK\alpha | H_{ppc} | IMK'\alpha' \rangle = \langle IMK\alpha | \frac{\hbar^2}{2\Im} \left(j_p^+ j_n^- - j_p^- j_n^+ \right) | IMK'\alpha' \rangle = \frac{\hbar^2}{2\Im} \delta_{KK'} \langle K\alpha | j_p^+ j_n^- + j_p^- j_n^+ | K'\alpha' \rangle$$

$$= \frac{\hbar^2}{2\Im} \begin{bmatrix} \delta_{KK'\neq 0} \begin{Bmatrix} \delta_{k_p k'_p+1} \delta_{k_n k'_n-1} \langle \rho_p k_p | j_p^+ | \rho'_p k'_p \rangle \langle \rho_n k_n | j_n^- | \rho'_n k'_n \rangle + \\ \delta_{k_p k'_p-1} \delta_{k_n k'_n+1} \langle \rho_p k_p | j_p^- | \rho'_p k'_p \rangle \langle \rho_n k_n | j_n^+ | \rho'_n k'_n \rangle \end{Bmatrix} + \\ \delta_{KK'=0} \delta_{J_\alpha J_{\alpha'}} \begin{Bmatrix} \delta_{kk'+1} \langle \rho_p k | j_p^+ | \rho'_p k' \rangle \langle \rho_n - k | j_n^- | \rho'_n - k' \rangle + \\ \delta_{kk'-1} \langle \rho_p k | j_p^- | \rho'_p k' \rangle \langle \rho_n - k | j_n^+ | \rho'_n - k' \rangle + \\ \delta_{k\frac{1}{2}} (-1)^{I+1} \langle \rho_p k | j_p^+ | \rho'_p - k' \rangle \langle \rho_n - k | j_n^- | \rho'_n k' \rangle \end{Bmatrix} \end{bmatrix} \tag{16}$$

Matrix elements of irrotational term (eq.7)

$$\langle IMK\alpha | H_{irrot} | IMK'\alpha' \rangle = \langle IMK\alpha | \frac{\hbar^2}{2\Im} \left[\left(j_p^2 - j_{pz}^2 \right) + \left(j_n^2 - j_{nz}^2 \right) \right] | IMK'\alpha' \rangle$$

$$= \frac{\hbar^2}{2\Im} \begin{bmatrix} \delta_{KK'\neq 0} \left\{ \langle \delta_p k_p | j_p^2 - j_{pz}^2 | \delta'_p k'_p \rangle \langle \delta_n k_n | j_n^2 - j_{nz}^2 | \delta'_n k'_n \rangle \right\} + \\ \delta_{KK'=0} \left\{ \langle \delta_p k | j_p^2 - j_{pz}^2 | \delta'_p k' \rangle \langle \delta_n k | j_n^2 - j_{nz}^2 | \delta'_n k' \rangle \right\} \end{bmatrix} \tag{17}$$

Matrix elements of residual *n-p* interaction (V_{np})

$$\langle IMK\alpha | V_{np} | IMK'\alpha' \rangle$$

$$= \delta_{KK'} \begin{bmatrix} \delta_{KK'\neq 0} \left(\langle K\alpha | V_{np} | K'\alpha' \rangle \right) + \\ \delta_{KK'=0} \delta_{J_\alpha J_{\alpha'}} \begin{pmatrix} \langle \rho_p k | \langle \rho_n - k | V_{np} | \rho'_p k' \rangle | \rho'_n - k' \rangle + \\ (-1)^{I+1} \langle \rho_p k | \langle \rho_n - k | V_{np} | \rho'_p - k' \rangle | \rho'_n k' \rangle \end{pmatrix} \end{bmatrix} \tag{18}$$

In this formulation of TQPRM, residual *n-p* interaction contributes only through diagonal matrix elements although in principle the non diagonal contribution should be included.

Matrix Elements for the various cases of couplings:

The various coupling case must be considered to formulate the total Hamiltonian. There are 9 different possibilities of coupling of projection of particle angular momenta (k_p, k_n) and are given below:

$$\begin{bmatrix} \left(K = k_p + k_n, K' = k'_p + k'_n\right) & (A) \\ \begin{pmatrix} K = k_p + k_n, K' = k'_p - k'_n \\ K = k_p + k_n, K' = k'_n - k'_p \end{pmatrix} & (B) \\ \begin{pmatrix} K = k_p - k_n, K' = k'_p + k'_n \\ K = k_n + k_p, K' = k'_p - k'_n \end{pmatrix} & (C) \\ \begin{pmatrix} K = k_p - k_n, K' = k'_p - k'_n \\ K = k_n - k_p, K' = k'_n - k'_p \\ K = k_p - k_n, K' = k'_n - k'_p \\ K = k_n - k_p, K' = k'_p - k'_n \end{pmatrix} & (D) \end{bmatrix} \tag{19}$$

To make representations of matrix elements of Coriolis coupling and particle-particle coupling in compact form, we re-define various couplings as

$$\begin{bmatrix} \sigma + = k_p + k_n = k_n + k_p \\ \sigma' + = k'_p + k'_n = k'_n + k'_p \\ \sigma - = k_p - k_n = k_n - k_p \\ \sigma' - = k'_p - k'_n = k'_n - k'_p \end{bmatrix} \tag{20}$$

Contribution of the Rotor Particle Coupling (RPC) terms for various cases of couplings

$$\left\langle IMK\alpha \left| H_{cor} \right| IMK'\alpha' \right\rangle = \left\langle IMK\alpha \left| -\frac{\hbar^2}{2\Im}\left(I_+ j_- - I_- j_+\right) \right| IMK'\alpha' \right\rangle$$

$$
= -\frac{\hbar^2}{2\Im}\left[\delta_{KK'-1}\sqrt{\left(I+K'\right)\left(I-K'+1\right)}\left\{\begin{array}{l}
\delta_{\sigma+}\delta_{\sigma'+}\left(\begin{array}{l}\left\langle\rho_p k'_p\left|j_p^+\right|\rho_p k_p\right\rangle\delta_{k'_p k_{p+1}}\delta_{\rho_n\rho'_n}\delta_{k_n k'_n}\\ \left\langle\rho_n k'_n\left|j_n^+\right|\rho_n k_n\right\rangle\delta_{k'_n k_{n+1}}\delta_{\rho_p\rho'_p}\delta_{k_p k'_p}\end{array}\right)+\\
\delta_{\sigma-}\delta_{\sigma'+}\left(\begin{array}{l}\left\langle\rho_p k'_p\left|j_p^+\right|\rho_p - k_p\right\rangle\delta_{k_p\frac{1}{2}}\delta_{k'_p\frac{1}{2}}\delta_{\rho_n\rho'_n}\delta_{k_n k'_n}\\ \left\langle\rho_n k'_n\left|j_n^+\right|\rho_n - k_n\right\rangle\delta_{k_n\frac{1}{2}}\delta_{k'_n\frac{1}{2}}\delta_{\rho_p\rho'_p}\delta_{k_p k'_p}\end{array}\right)+\\
\delta_{\sigma-}\delta_{\sigma'-}\left(\begin{array}{l}\left\langle\rho'_p k'_p\left|j_p^+\right|\rho_p k_p\right\rangle\delta_{k'_p k_{p+1}}\delta_{\rho_n\rho'_n}\delta_{k_n k'_n}\\ \left\langle\rho'_n k'_n\left|j_n^+\right|\rho_n k_n\right\rangle\delta_{k'_n k_{n+1}}\delta_{\rho_p\rho'_p}\delta_{k_p k'_p}\end{array}\right)
\end{array}\right\}+\right.
$$

$$
+\delta_{KK'+1}\sqrt{\left(I-K'\right)\left(I+K'+1\right)}\left\{\begin{array}{l}
\delta_{\sigma+}\delta_{\sigma'+}\left(\begin{array}{l}\left\langle\rho_p k_p\left|j_p^+\right|\rho'_p k'_p\right\rangle\delta_{k_p k'_{p+1}}\delta_{\rho_n\rho'_n}\delta_{k_n k'_n}\\ \left\langle\rho_n k_n\left|j_n^+\right|\rho'_n k'_n\right\rangle\delta_{k_n k'_{n+1}}\delta_{\rho_p\rho'_p}\delta_{k_p k'_p}\end{array}\right)+\\
\delta_{\sigma'-}\delta_{\sigma+}\left(\begin{array}{l}\left\langle\rho_p k_p\left|j_p^+\right|\rho'_p - k'_p\right\rangle\delta_{k'_p\frac{1}{2}}\delta_{k_p\frac{1}{2}}\delta_{\rho_n\rho'_n}\delta_{k_n k'_n}\\ \left\langle\rho_n k_n\left|j_n^+\right|\rho'_n - k'_n\right\rangle\delta_{k'_n\frac{1}{2}}\delta_{k_n\frac{1}{2}}\delta_{\rho_p\rho'_p}\delta_{k_p k'_p}\end{array}\right)+\\
\delta_{\sigma'-}\delta_{\sigma-}\left(\begin{array}{l}\left\langle\rho_p k_p\left|j_p^+\right|\rho'_p k'_p\right\rangle\delta_{k_p k'_{p+1}}\delta_{\rho_n\rho'_n}\delta_{k_n k'_n}\\ \left\langle\rho_n k_n\left|j_n^+\right|\rho'_n k'_n\right\rangle\delta_{k_n k'_{n+1}}\delta_{\rho_p\rho'_p}\delta_{k_p k'_p}\end{array}\right)
\end{array}\right\}+
$$

$$
+\delta_{K=1}\delta_{K'=0}\frac{1}{2\sqrt{2}}\left[\left(1+\left(-1\right)^I J_{\alpha'}\right)^2\right]\sqrt{I\left(I+1\right)}\frac{1}{\sqrt{2}}\left\{\begin{array}{l}
\delta_{\sigma+}\delta_{\sigma'-}\left(\begin{array}{l}\left\langle\rho_n\frac{1}{2}\right|j_n^+\left|\rho'_n - \frac{1}{2}\right\rangle\delta_{k_n\frac{1}{2}}\delta_{\rho_p\rho'_p}\delta_{k_p\frac{1}{2}}\delta_{k\frac{1}{2}}+\\ \left(-1\right)^{I+1}\left\langle\rho_n\frac{1}{2}\right|j_p^+\left|\rho'_p - \frac{1}{2}\right\rangle\delta_{k_n\frac{1}{2}}\delta_{\rho_n\rho'_n}\delta_{k_p\frac{1}{2}}\delta_{k\frac{1}{2}}\end{array}\right)+\\
\delta_{\sigma-}\delta_{\sigma'-}\left(\begin{array}{l}\left\langle\rho_p k_p\left|j_p^+\right|\rho'_n k'\right\rangle\delta_{k_p k'+1}\delta_{\rho_n\rho'_n}\delta_{k_n k'_n}+\\ \left(-1\right)^{I+1}\left\langle\rho_n k_n\left|j_n^+\right|\rho'_n k'\right\rangle\delta_{k_n k'+1}\delta_{\rho_p\rho'_p}\delta_{k_p k'}\end{array}\right)
\end{array}\right\}+
$$

$$+\delta_{K'=1}\delta_{K=0}\frac{1}{2\sqrt{2}}\left[\left(1+(-1)^{I}J_{\alpha}\right)^{2}\right]\sqrt{I(I+1)}\frac{1}{\sqrt{2}}\left\{\begin{array}{l}\delta_{\sigma+}\delta_{\sigma-}\left(\begin{array}{l}\left\langle\rho_n'\frac{1}{2}\middle|j_n^{+}\middle|\rho_n-\frac{1}{2}\right\rangle\delta_{k_n'\frac{1}{2}}\delta_{\rho_p'\rho_p}\delta_{k_p'\frac{1}{2}k\frac{1}{2}}+\\(-1)^{I+1}\left\langle\rho_n'\frac{1}{2}\middle|j_p^{+}\middle|\rho_p-\frac{1}{2}\right\rangle\delta_{k_n'\frac{1}{2}}\delta_{\rho_n'\rho_n}\delta_{k_p'\frac{1}{2}k\frac{1}{2}}\end{array}\right)+\\\delta_{\sigma'-}\delta_{\sigma-}\left(\begin{array}{l}\left\langle\rho_p'k_p'\middle|j_p^{+}\middle|\rho_nk\right\rangle\delta_{k_p'k+1}\delta_{\rho_n\rho_n'}\delta_{k_n'k_n}+\\(-1)^{I+1}\left\langle\rho_n'k_n'\middle|j_n^{+}\middle|\rho_n-k\right\rangle\delta_{k_n'k+1}\delta_{\rho_p\rho_p'}\delta_{k_p'k}\end{array}\right)\end{array}\right\}\qquad(21)$$

Contribution of the Particle-Particle Coupling (PPC) terms for various cases of couplings

$$\left\langle IMK\alpha\middle|H_{ppc}\middle|IMK'\alpha'\right\rangle=\left\langle IMK\alpha\middle|\frac{\hbar^2}{2\Im}\left(j_p^{+}j_n^{-}-j_p^{-}j_n^{+}\right)\middle|IMK'\alpha'\right\rangle$$

$$=\delta_{KK'}\frac{\hbar^2}{2\Im}\left[\begin{array}{l}\left\{\delta_{KK'=0}\delta_{\alpha\alpha'}\delta_{\sigma\sigma'}\delta_{\sigma-}(-1)^{I+1}\left\langle\rho_pk\middle|j_p^{+}\middle|\rho_p-k\right\rangle\left\langle\rho_nk\middle|j_n^{+}\middle|\rho_n-k\right\rangle\delta_{k\frac{1}{2}}\right\}+\\\\\left\{\delta_{KK'\neq0}\delta_{\sigma+\sigma'+}\left(\begin{array}{l}\left\langle\rho_pk_p\middle|j_p^{+}\middle|\rho_p'k_p'\right\rangle\left\langle\rho_n'k_n'\middle|j_n^{+}\middle|\rho_nk_n\right\rangle\delta_{k_pk_p'+1}\delta_{k_nk_n'+1}+\\\left\langle\rho_p'k_p'\middle|j_p^{+}\middle|\rho_pk_p\right\rangle\left\langle\rho_nk_n\middle|j_n^{+}\middle|\rho_n'k_n'\right\rangle\delta_{k_pk_p'+1}\delta_{k_nk_n'+1}\end{array}\right)\right\}+\\\\\left\{\delta_{KK'\neq0}\delta_{\sigma+\sigma'-}\left(\begin{array}{l}\left\langle\rho_p'k_p'\middle|j_p^{+}\middle|\rho_pk_p\right\rangle\left\langle\rho_nk_n\middle|j_n^{+}\middle|\rho_n'-k_n'\right\rangle\delta_{k_pk_p'+1}\delta_{k_n\frac{1}{2}}\delta_{k_n'\frac{1}{2}}+\\\left\langle\rho_pk_p\middle|j_p^{+}\middle|\rho_pk_p'\right\rangle\left\langle\rho_n'k_n'\middle|j_n^{+}\middle|\rho_nk_n\right\rangle\delta_{k_nk_n'+1}\delta_{k_p\frac{1}{2}}\delta_{k_p'\frac{1}{2}}\end{array}\right)\right\}+\\\\\left\{\delta_{KK'\neq0}\delta_{\sigma-\sigma'+}\left(\begin{array}{l}\left\langle\rho_pk_p\middle|j_p^{+}\middle|\rho_p'k_p'\right\rangle\left\langle\rho_nk_n\middle|j_n^{+}\middle|\rho_n'-k_n'\right\rangle\delta_{k_pk_p'+1}\delta_{k_n\frac{1}{2}}\delta_{k_n'\frac{1}{2}}+\\\left\langle\rho_pk_p\middle|j_p^{+}\middle|\rho_p-k_p'\right\rangle\left\langle\rho_nk_n\middle|j_n^{+}\middle|\rho_n'k_n'\right\rangle\delta_{k_nk_n'+1}\delta_{k_p\frac{1}{2}}\delta_{k_p'\frac{1}{2}}\end{array}\right)\right\}+\end{array}\right.$$

$$\left\{\delta_{KK'=0}\delta_{\sigma-\sigma'-}\left(\begin{array}{l}-\left\langle\rho_p k\left|j_p^+\right|\rho'_p k'\right\rangle\left\langle\rho_n k\left|j_n^+\right|\rho'_n k\right\rangle\delta_{kk'+1}+\\-\left\langle\rho'_p k'\left|j_p^+\right|\rho_p k\right\rangle\left\langle\rho'_n k\left|j_n^+\right|\rho_n k\right\rangle\delta_{k'k+1}+\\(-1)^{I+1}\left[\left\langle\rho_p k\left|j_p^+\right|\rho'_p-k'\right\rangle\left\langle\rho_n k\left|j_n^+\right|\rho'_n-k'\right\rangle\delta_{k_p\frac{1}{2}}\delta_{k_n\frac{1}{2}}\delta_{k_p\frac{1}{2}}\delta_{k_n\frac{1}{2}}\delta_{k\frac{1}{2}}\right]\end{array}\right)\right\} \tag{22}$$

In order to simplify the total Hamiltonian, we define following quantities as:

$$\left[\begin{array}{l}\alpha+=\left(k_p=k_n\right)=\left(k'_p=k'_n\right)\\\alpha-=\left(-k_p=-k_n\right)=\left(-k'_p=-k'_n\right)\end{array}\right] \tag{23}$$

$$\left[\begin{array}{l}E_{\alpha+}=\varepsilon\rho_n+\varepsilon\rho_p+\left\langle\rho_p k_p\left|\left\langle\rho_n k_n\left|V_{np}\right|\rho_p k_p\right\rangle\right|\rho_n k_n\right\rangle+\left\langle K\alpha+\left|H_{irrot}\right|K\alpha+\right\rangle\\E_{\alpha+}=\varepsilon\rho_n+\varepsilon\rho_p+\left\langle\rho_p k_p\left|\left\langle\rho_n-k_n\left|V_{np}\right|\rho_p-k_p\right\rangle\right|\rho_n-k_n\right\rangle+\left\langle K\alpha-\left|H_{irrot}\right|K\alpha-\right\rangle\\C_\alpha=\left\langle\rho_p k_p\left|\left\langle\rho_n-k_n\left|V_{np}\right|\rho_p-k_p\right\rangle\right|\rho_n k_n\right\rangle\end{array}\right] \tag{24}$$

The band head energies are denoted by $E_{\alpha+}, E_{\alpha-}$ which corresponds to band head of GM doublets (K_+, K_-). The Newby shift is denoted by C_α , which absorbs the diagonal contributions of residual *n-p* interaction. The $\left\langle K\alpha+\left|H_{irrot}\right|K\alpha+\right\rangle, \left\langle K\alpha-\left|H_{irrot}\right|K\alpha-\right\rangle$ are the diagonal contributions of irrotational (H_{irrot}) term.

Using the definitions given by equations (23, 24) in equations 14, 18, 21, and 22, the total Hamiltonian can be written as:

$$\left\langle IMK\alpha\sigma\left|H_{tot}\right|IMK'\alpha'\sigma'\right\rangle=\left\langle IMK\alpha\sigma\left|H_{rot}^0+H_{cor}+H_{ppc}+H_{irrot}+V_{np}\right|IMK'\alpha'\sigma'\right\rangle$$

$$= \delta_{KK'} \Bigg[\delta_{\alpha\alpha'}\delta_{\sigma\sigma'} \begin{Bmatrix} E_{\alpha\sigma} + \dfrac{\hbar^2}{2\Im}\left[I\left(I+1\right) - K^2 \right] + \delta_{KK'=0}\delta_{\sigma-}\left(-1\right)^{I+1} C_\alpha + \\ \delta_{KK'=0}\delta_{\sigma-}\left(-1\right)^{I+1} \dfrac{\hbar^2}{2\Im} \left\langle \rho_p k \middle| j_p^+ \middle| \rho_p - k \right\rangle \left\langle \rho_n k \middle| j_n^+ \middle| \rho_n - k \right\rangle \delta_{k\frac{1}{2}} \end{Bmatrix} +$$

$$\left\{ \delta_{KK'\neq 0}\delta_{\sigma+\sigma'+} \frac{\hbar^2}{2\Im} \begin{pmatrix} \left\langle \rho_p k_p \middle| j_p^+ \middle| \rho'_p k'_p \right\rangle \left\langle \rho'_n k'_n \middle| j_n^+ \middle| \rho_n k_n \right\rangle \delta_{k_p k'_p+1}\delta_{k_n k'_n+1} + \\ \left\langle \rho'_p k'_p \middle| j_p^+ \middle| \rho_p k_p \right\rangle \left\langle \rho_n k_n \middle| j_n^+ \middle| \rho'_n k'_n \right\rangle \delta_{k_p k'_p+1}\delta_{k_n k'_n+1} \end{pmatrix} \right\} +$$

$$\left\{ \delta_{KK'\neq 0}\delta_{\sigma+\sigma'-} \frac{\hbar^2}{2\Im} \begin{pmatrix} \left\langle \rho'_p k'_p \middle| j_p^+ \middle| \rho_p k_p \right\rangle \left\langle \rho_n k_n \middle| j_n^+ \middle| \rho'_n - k'_n \right\rangle \delta_{k_p k'_p+1}\delta_{k_n \frac{1}{2}}\delta_{k'_n \frac{1}{2}} + \\ \left\langle \rho_p k_p \middle| j_p^+ \middle| \rho_p k'_p \right\rangle \left\langle \rho'_n k'_n \middle| j_n^+ \middle| \rho_n k_n \right\rangle \delta_{k_n k'_n+1}\delta_{k_p \frac{1}{2}}\delta_{k'_p \frac{1}{2}} \end{pmatrix} \right\} +$$

$$\left\{ \delta_{KK'\neq 0}\delta_{\sigma-\sigma'+} \frac{\hbar^2}{2\Im} \begin{pmatrix} \left\langle \rho_p k_p \middle| j_p^+ \middle| \rho'_p k'_p \right\rangle \left\langle \rho_n k_n \middle| j_n^+ \middle| \rho'_n - k'_n \right\rangle \delta_{k_p k'_p+1}\delta_{k_n \frac{1}{2}}\delta_{k'_n \frac{1}{2}} + \\ \left\langle \rho_p k_p \middle| j_p^+ \middle| \rho_p - k'_p \right\rangle \left\langle \rho_n k_n \middle| j_n^+ \middle| \rho'_n k'_n \right\rangle \delta_{k_n k'_n+1}\delta_{k_p \frac{1}{2}}\delta_{k'_p \frac{1}{2}} \end{pmatrix} \right\} +$$

$$\left\{ \delta_{KK'=0}\delta_{\sigma-\sigma'-} \frac{\hbar^2}{2\Im} \begin{pmatrix} -\left\langle \rho_p k \middle| j_p^+ \middle| \rho'_p k' \right\rangle \left\langle \rho_n k \middle| j_n^+ \middle| \rho'_n k \right\rangle \delta_{kk'+1} + \\ -\left\langle \rho'_p k' \middle| j_p^+ \middle| \rho_p k \right\rangle \left\langle \rho'_n k \middle| j_n^+ \middle| \rho_n k \right\rangle \delta_{k'k+1} + \\ \left(-1\right)^{I+1} \left[\left\langle \rho_p k \middle| j_p^+ \middle| \rho'_p - k' \right\rangle \left\langle \rho_n k \middle| j_n^+ \middle| \rho'_n - k' \right\rangle \delta_{k_p \frac{1}{2}}\delta_{k_n \frac{1}{2}}\delta_{k_p \frac{1}{2}}\delta_{k_n \frac{1}{2}}\delta_{k\frac{1}{2}} \right] \end{pmatrix} \right\} +$$

$$\delta_{KK'-1}\left(-\frac{\hbar^2}{2\Im}\right)\sqrt{\left(I+K'\right)\left(I-K'+1\right)} \left\{ \begin{array}{l} \delta_{\sigma+}\delta_{\sigma'+} \begin{pmatrix} \left\langle \rho_p k'_p \middle| j_p^+ \middle| \rho_p k_p \right\rangle \delta_{k'_p k_{p+1}}\delta_{\rho_n \rho'_n}\delta_{k_n k'_n} \\ \left\langle \rho_n k'_n \middle| j_n^+ \middle| \rho_n k_n \right\rangle \delta_{k'_n k_{n+1}}\delta_{\rho_p \rho'_p}\delta_{k_p k'_p} \end{pmatrix} + \\ \delta_{\sigma-}\delta_{\sigma'+} \begin{pmatrix} \left\langle \rho_p k'_p \middle| j_p^+ \middle| \rho_p - k_p \right\rangle \delta_{k_p \frac{1}{2}}\delta_{k'_p \frac{1}{2}}\delta_{\rho_n \rho'_n}\delta_{k_n k'_n} \\ \left\langle \rho_n k'_n \middle| j_n^+ \middle| \rho_n - k_n \right\rangle \delta_{k_n \frac{1}{2}}\delta_{k'_n \frac{1}{2}}\delta_{\rho_p \rho'_p}\delta_{k_p k'_p} \end{pmatrix} + \\ \delta_{\sigma-}\delta_{\sigma'-} \begin{pmatrix} \left\langle \rho'_p k'_p \middle| j_p^+ \middle| \rho_p k_p \right\rangle \delta_{k'_p k_{p+1}}\delta_{\rho_n \rho'_n}\delta_{k_n k'_n} \\ \left\langle \rho'_n k'_n \middle| j_n^+ \middle| \rho_n k_n \right\rangle \delta_{k'_n k_{n+1}}\delta_{\rho_p \rho'_p}\delta_{k_p k'_p} \end{pmatrix} \end{array} \right\} +$$

$$+\delta_{K=1}\delta_{K'=0}\frac{1}{2\sqrt{2}}\left(-\frac{\hbar^2}{2\Im}\right)\left[\left(1+\left(-1\right)^{I}J_{\alpha'}\right)^2\right]\sqrt{I\left(I+1\right)}\frac{1}{\sqrt{2}}\left\{\begin{array}{l}\delta_{\sigma+}\delta_{\sigma'-}\left(\begin{array}{l}\left\langle\rho_n\frac{1}{2}\middle|j_n^+\middle|\rho_n'-\frac{1}{2}\right\rangle\delta_{k_n\frac{1}{2}}\delta_{\rho_p\rho_p'}\delta_{k_p\frac{1}{2}}\delta_{k\frac{1}{2}}+\\ \left(-1\right)^{I+1}\left\langle\rho_n\frac{1}{2}\middle|j_p^+\middle|\rho_p'-\frac{1}{2}\right\rangle\delta_{k_n\frac{1}{2}}\delta_{\rho_n\rho_n'}\delta_{k_p\frac{1}{2}}\delta_{k\frac{1}{2}}\end{array}\right)+\\ \delta_{\sigma-}\delta_{\sigma'-}\left(\begin{array}{l}\left\langle\rho_pk_p\middle|j_p^+\middle|\rho_n'k'\right\rangle\delta_{k_pk'+1}\delta_{\rho_n\rho_n'}\delta_{k_nk_n'}+\\ \left(-1\right)^{I+1}\left\langle\rho_nk_n\middle|j_n^+\middle|\rho_n'k'\right\rangle\delta_{k_nk'+1}\delta_{\rho_p\rho_p'}\delta_{k_pk'}\end{array}\right)\end{array}\right\}+$$

$$+\delta_{KK'+1}\left(-\frac{\hbar^2}{2\Im}\right)\sqrt{\left(I-K'\right)\left(I+K'+1\right)}\left\{\begin{array}{l}\delta_{\sigma+}\delta_{\sigma'+}\left(\begin{array}{l}\left\langle\rho_pk_p\middle|j_p^+\middle|\rho_p'k_p'\right\rangle\delta_{k_pk_{p+1}'}\delta_{\rho_n\rho_n'}\delta_{k_nk_n'}\\ \left\langle\rho_nk_n\middle|j_n^+\middle|\rho_n'k_n'\right\rangle\delta_{k_nk_{n+1}'}\delta_{\rho_p\rho_p'}\delta_{k_pk_p'}\end{array}\right)+\\ \delta_{\sigma'-}\delta_{\sigma+}\left(\begin{array}{l}\left\langle\rho_pk_p\middle|j_p^+\middle|\rho_p'-k_p'\right\rangle\delta_{k_p'\frac{1}{2}}\delta_{k_p\frac{1}{2}}\delta_{\rho_n\rho_n'}\delta_{k_nk_n'}\\ \left\langle\rho_nk_n\middle|j_n^+\middle|\rho_n'-k_n'\right\rangle\delta_{k_n'\frac{1}{2}}\delta_{k_n\frac{1}{2}}\delta_{\rho_p\rho_p'}\delta_{k_pk_p'}\end{array}\right)+\\ \delta_{\sigma'-}\delta_{\sigma-}\left(\begin{array}{l}\left\langle\rho_pk_p\middle|j_p^+\middle|\rho_p'k_p'\right\rangle\delta_{k_pk_{p+1}'}\delta_{\rho_n\rho_n'}\delta_{k_nk_n'}\\ \left\langle\rho_nk_n\middle|j_n^+\middle|\rho_n'k_n'\right\rangle\delta_{k_nk_{n+1}'}\delta_{\rho_p\rho_p'}\delta_{k_pk_p'}\end{array}\right)\end{array}\right\}+$$

$$+\delta_{K'=1}\delta_{K=0}\frac{1}{2\sqrt{2}}\left(-\frac{\hbar^2}{2\Im}\right)\left[\left(1+\left(-1\right)^{I}J_{\alpha}\right)^2\right]\sqrt{I\left(I+1\right)}\frac{1}{\sqrt{2}}\left\{\begin{array}{l}\delta_{\sigma+}\delta_{\sigma-}\left(\begin{array}{l}\left\langle\rho_n'\frac{1}{2}\middle|j_n^+\middle|\rho_n-\frac{1}{2}\right\rangle\delta_{k_n'\frac{1}{2}}\delta_{\rho_p'\rho_p}\delta_{k_p'\frac{1}{2}}\delta_{k\frac{1}{2}}+\\ \left(-1\right)^{I+1}\left\langle\rho_n'\frac{1}{2}\middle|j_p^+\middle|\rho_p-\frac{1}{2}\right\rangle\delta_{k_n'\frac{1}{2}}\delta_{\rho_n'\rho_n}\delta_{k_p'\frac{1}{2}}\delta_{k\frac{1}{2}}\end{array}\right)+\\ \delta_{\sigma'-}\delta_{\sigma-}\left(\begin{array}{l}\left\langle\rho_p'k_p'\middle|j_p^+\middle|\rho_nk\right\rangle\delta_{k_p'k+1}\delta_{\rho_n\rho_n'}\delta_{k_n'k_n}+\\ \left(-1\right)^{I+1}\left\langle\rho_n'k_n'\middle|j_n^+\middle|\rho_n-k\right\rangle\delta_{k_n'k+1}\delta_{\rho_p\rho_p'}\delta_{k_p'k}\end{array}\right)\end{array}\right\}\Bigg]$$

(25)

Diagonalization of total Hamiltonian matrix (H_{tot}) for each value of angular momentum I gives us the theoretical value of energy ($E_{theor}\left(I,\alpha\sigma\right)$) for all the bands built over 2qp configurations ($\left|K\alpha\sigma\right\rangle$) present in the given basis.

XI APPLICATIONS OF PROPOSED STUDY

- The present study will be useful in fixing the universal cause for signature effects observed in two quasi-particle rotational bands of odd-odd nuclei.
- The various corrections such as Coriolis and two state mixing will give better estimates of band-head energies.
- The theoretical estimates of the residual interactions will improve our understanding of *n-p* interaction.

X WORK DONE SO FAR

- Theoretical formulation except residual interaction has been completed.
- Extensive literature review in the mass region A=160 has been done. The number of 2qp bands exhibiting signature effects and also various theoretical approaches existing in literature for the explanation of these signature effects in odd-odd nuclei has been extracted.

XI TENTATIVE CHAPTER PLAN FOR THESIS

Chapter I: Introduction.

Chapter II: Complete theoretical model for the calculation of band head energies of 2qp bands.

Chapter III: Formulation of TQPRM with inclusion of theoretical *n-p* interaction (V_{np}).

Chapter IV Application of TQPRM for the explanation of observed signature effects in odd-odd nuclei.

Chapter V: Effect of Coriolis mixing on band head energies of 2qp bands.

Chapter VI: Conclusions.

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